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First-Order Model of Symmetrical Six-Port Microstrip Ring Coupler

S. P. Yeo and C. L. Lau

***Abstract*—This paper describes, in brief, how the simple eigenmode approach can be utilized to develop a first-order model that yields explicit ready-to-use formulas for predicting the performance characteristics of a symmetrical six-port microstrip ring coupler. Prototype tests conducted over the 2–5 GHz frequency range show the agreement between the predicted and measured values of the coupler's scattering coefficients to be within ±0.05 for magnitude and ±10° for phase.**

I. Introduction

The symmetrical six-port junction (Fig. 1) has over the past few years been attracting the attention of various researchers. Riblet *et al.* [1] designed one for use as a five-way equal power divider, while Judah *et al.* [2] and Yeo *et al.* [3] showed how it could also be utilized in a six-port reflectometer setup.

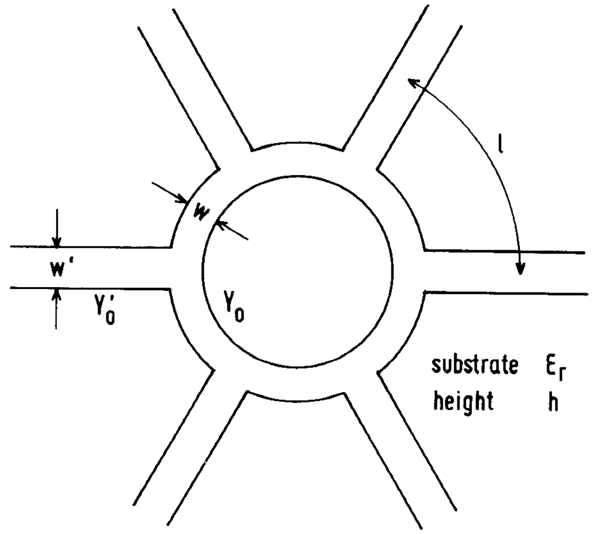

Fig. 1. Symmetrical six-port microstrip ring coupler.

Thus far, satisfactory models to predict the characteristics of the stripline [1] and waveguide [4] versions of the symmetrical six-port junction have been reported in the literature. Our objective in this paper, therefore, is to extend the investigation to include the analysis of the microstrip version. Actually, a model of one such microstrip coupler has already been put forward by Judah *et al.* [2]; however, their circuit topology is more complicated than that of Fig. 1 because they inserted an additional node at the hub of the structure (thereby rendering it, for purposes of analysis, effectively a seven-port instead of a six-port). In contrast, we chose to retain the original simplicity of the ring layout in Fig. 1 so as to obviate the necessity of performing a seven-port to six-port circuit reduction (as Judah *et al.* [2] had to do).

II. Theory

There are two approaches that we can take in the formulation of our model: eigenmode or noneigenmode. The latter has the problem of yielding rather long and unwieldy expressions, although, as one referee has pointed out, it does offer flexibility for studying nonsymmetries in the circuit. The method used in this paper is based on the eigenmode approach since this, as has been demonstrated in previous analyses of the symmetrical N-port junctions [4]–[6], yields simple explicit formulas that can be readily used for design work.

Assuming that the curvature of the central ring line can be ignored (as Cullen *et al.* [7] and Judah *et al.* [2] did in their analyses), we are able to represent the circuit connections between any three consecutive ports $k-1, k, k+1$ by the equivalent transmission-line model of Fig. 2, where, for the mth eigenmode,

$$v_k = v_{k-1}\exp\left(-j\frac{m\pi}{3}\right) = v_{k+1}\exp\left(j\frac{m\pi}{3}\right). \tag{1}$$

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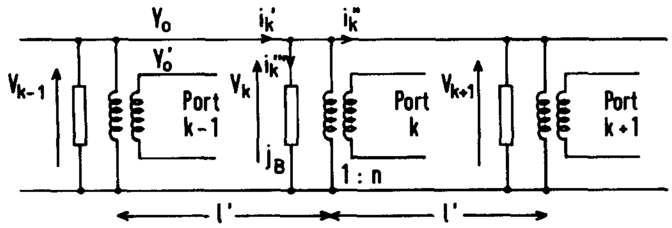


Fig. 2. Equivalent transmission-line model for any three consecutive ports $k-1, k, k+1$.

An analysis of the circuit of Fig. 2 then yields the following expressions for the current components at node k:

$$i_k' = -\frac{jY_0 v_k}{\sin \beta l'}\left[\exp\left(j\frac{m\pi}{3}\right) - \cos \beta l'\right] \tag{2}$$

$$i_k'' = \frac{jY_0 v_k}{\sin \beta l'}\left[\exp\left(-j\frac{m\pi}{3}\right) - \cos \beta l'\right] \tag{3}$$

$$i_k''' = jBv_k. \tag{4}$$

The combination of (2)–(4) thus permits us to derive the eigenadmittance (for the mth eigenmode) looking into any of the six ports:

$$\hat{Y}_m = \frac{j}{n^2}\left\{\frac{2Y_0}{\sin \beta l'}\left[\cos\left(\frac{m\pi}{3}\right) - \cos \beta l'\right] + B\right\}. \tag{5}$$

(Expressions for the phase constant β of the central ring line, the length correction $\Delta l = l - l'$ for the distance between adjacent ports, the shunt susceptance jB present at the junction, the turns ratio $n:1$ of the transformer, and the characteristic admittances Y_0 and Y_0' of the central ring line and the six arm lines, respectively, can be found in standard references, such as [8].)

From (5), we can compute four distinct eigenadmittances, $\hat{Y}_0, \hat{Y}_1, \hat{Y}_2, \hat{Y}_3$ (corresponding to the $m = 0,1,2,3$ eigenmodes supported by the symmetrical six-port structure of Fig. 1), and thereafter obtain the scattering matrix of the coupler:

$$S = \begin{bmatrix} \gamma & \alpha & \delta & \tau & \delta & \alpha \\ \alpha & \gamma & \alpha & \delta & \tau & \delta \\ \delta & \alpha & \gamma & \alpha & \delta & \tau \\ \tau & \delta & \alpha & \gamma & \alpha & \delta \\ \delta & \tau & \delta & \alpha & \gamma & \alpha \\ \alpha & \delta & \tau & \delta & \alpha & \gamma \end{bmatrix} \tag{6}$$

via the following formulas [1]:

$$\gamma = \frac{1}{6}(\lambda_0 + 2\lambda_1 + 2\lambda_2 + \lambda_3) \tag{7}$$

$$\tau = \frac{1}{6}(\lambda_0 - 2\lambda_1 + 2\lambda_2 - \lambda_3) \tag{8}$$

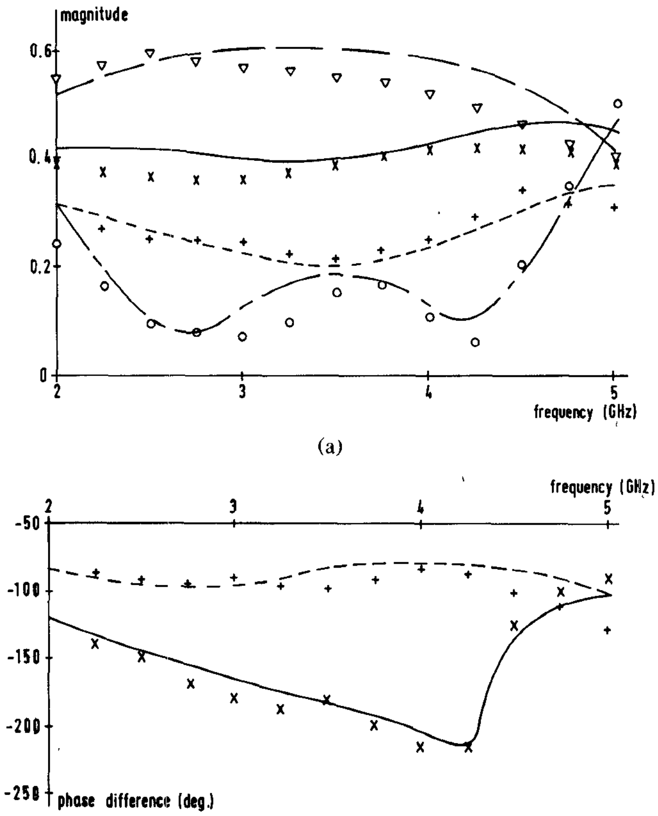
$$\alpha = \frac{1}{6}(\lambda_0 + \lambda_1 - \lambda_2 - \lambda_3) \tag{9}$$

$$\delta = \frac{1}{6}(\lambda_0 - \lambda_1 - \lambda_2 + \lambda_3) \tag{10}$$

where the eigenreflection coefficients λ_m are given by

$$\lambda_m = \frac{Y_0' - \hat{Y}_m}{Y_0' + \hat{Y}_m} \quad \text{for } m = 0,1,2,3. \tag{11}$$

(As an aside, we would like to mention that, apart from the computation of the coupler's scattering coefficients, another possible application of the eigenadmittances generated from (5) is in the computation of the coupler's equivalent admittance [9] using the formula derived by Riblet *et al.* in [1].)



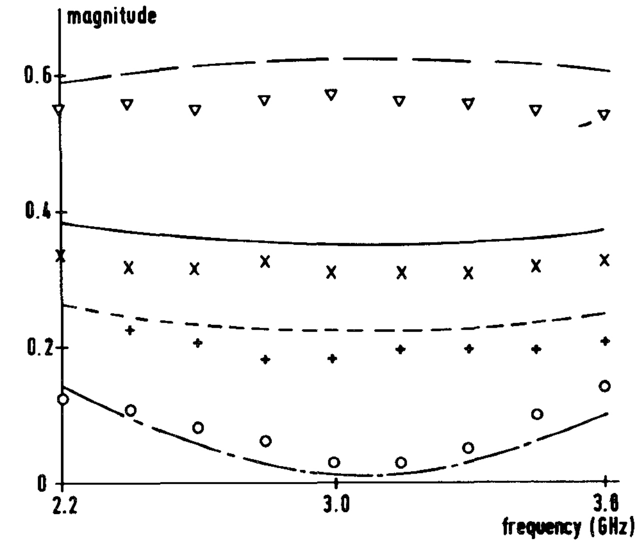
(b)

Fig. 3. Performance of prototype coupler #1: $w = 5.0$ mm, $l = 17.5$ mm, $w' = 6.4$ mm, $h = 2.3$ mm, $\epsilon_r = 2.5$. (a) Magnitudes of coupler's scattering coefficients:
$|\alpha|$: ——— theory, ∇∇∇ experiment;
$|\delta|$: ------- theory, + + + experiment;
$|\tau|$: ——— theory, × × × experiment;
$|\gamma|$: ---- theory, ∘ ∘ ∘ experiment.
(b) Phase differences between coupler's transmission coefficients:
$\arg(\delta/\alpha)$: --- theory, + + + experiment;
$\arg(\tau/\alpha)$: ——— theory, × × × experiment.

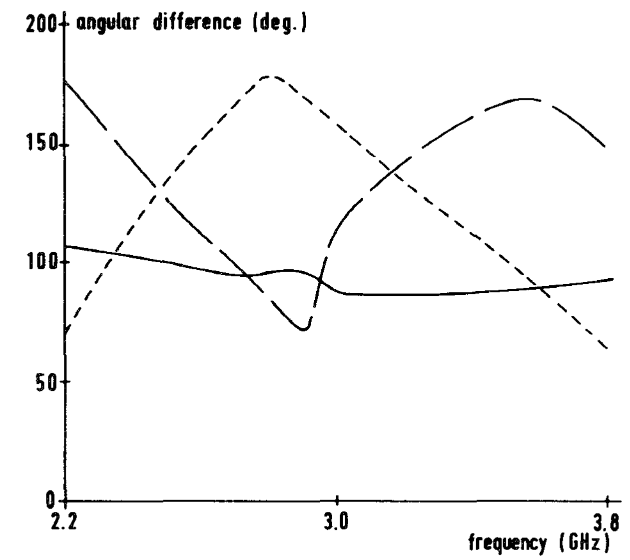
III. Results

The model developed in Section II (with dispersion effect [8] taken into account) is then implemented as a self-contained software package (written in Turbo Pascal Version 5.0)[1] for running on the IBM PC. Fig. 3 presents a typical plot of the frequency variations of γ, α, δ, and τ for the following parameter settings: $w = 5.0$ mm, $w' = 6.4$ mm, $l = 17.5$ mm, $h = 2.3$ mm, and $\epsilon_r = 2.5$. To verify the accuracy of the computed results, we fabricated prototype #1 (using the aforementioned dimensions) at the National University of Singapore and then sent it to the metrology laboratory at the Defense Science Organization (Singapore Ministry of Defense) for testing over the 2–5 GHz frequency range. When plotted alongside each other in Fig. 3, the measured and predicted results show, for most of the frequency range, reasonably close agreement with each

[1]Readers can obtain a complimentary copy of the software code by writing to the authors.



(a)



(b)

Fig. 4. Performance of prototype coupler #2: $w = 5.1$ mm, $l = 15.7$ mm, $w' = 7.8$ mm, $h = 2.3$ mm, $\epsilon_r = 2.5$ (a) Magnitudes of coupler's scattering coefficients:
$|\alpha|$: ——— theory, $\nabla\nabla\nabla$ experiment;
$|\delta|$: ------- theory, + + + experiment;
$|\tau|$: ——— theory, × × × experiment;
$|\gamma|$: ---- theory, ∘ ∘ ∘ experiment.
(b) Computed angular separations of q points [10] for coupler when used as a six-port reflectometer [2] (the DUT in this instance being a matched load):
--- $\arg(q_1/q_3)$;
——— $\arg(q_3/q_5)$;
--------- $\arg(q_5/q_1)$.

other—within ± 0.05 for magnitude and $\pm 10°$ for phase. It would thus appear that, similar to what Cullen *et al.* [7] and Judah *et al.* [2] had observed, neglecting the effects of the curvature of the central ring line does not seriously impair the utility of our computer model.

In [2], Judah *et al.* demonstrated that, for the special case when $|\gamma| = |\delta| = 0$ and $|\alpha| = |\tau| = 1/\sqrt{3}$, the symmetrical six-port coupler should in principle be well suited for use as part of a proposed six-port reflectometer setup. However, they did not proceed to predict how the resulting q points of the reflectometer would have varied when the coupler's scattering coefficients departed from their design values. To check on this matter, we present in Fig. 4 the scattering coefficients as well as the q-point distribution (using the additional formulas derived in [10] for the coupler when configured as a six-port reflectometer [2]) of another design, prototype #2, for which $w = 5.1$ mm, $w' = 7.8$ mm, $l = 15.7$ mm, $h = 2.3$ mm, and $\epsilon_r = 2.5$. Fig. 4(a) shows that, of the four scattering coefficients, only $|\gamma|$ and $|\alpha|$ meet the design specifications ($|\gamma| < 0.1$ and $|\alpha| - 1/\sqrt{3} < 0.05$ over the 2.4–3.8 GHz frequency range). Nevertheless, the resulting q points of Fig. 4(b) remain reasonably well spaced and the reflectometer should thus be capable of satisfactory operation over a 45% bandwidth.

IV. Conclusion

Instead of adopting other noneigenmode procedures (which tend to produce long and unwieldy expressions for symmetrical structures having more than, say, five ports), we have demonstrated in the present analysis how the simple eigenmode approach can be utilized to develop a first-order model that yields explicit formulas for predicting the performance characteristics of the symmetrical six-port microstrip ring coupler. Tests conducted on prototype couplers over the 2–5 GHz frequency range indicate that the agreement between theory and experiment is reasonably good (within ± 0.05 for magnitude and $\pm 10°$ for phase), thereby corroborating, for the range of parameter settings used, the validity of the assumptions which have been incorporated into the model.

As a final note, we wish to add that the theory outlined here can, if suitably reformulated in general terms for a symmetrical N-port microstrip coupler, also be extended in scope to include cases where $N > 6$.

Acknowledgment

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Properties of TE–TM Mode-Matching Techniques

Gian Guido Gentili

Abstract—**A line integral formulation of TE–TM mode-matching techniques for scattering problems in waveguides is described. The procedure is convenient from a computational point of view when the modes in the waveguides must be computed numerically. Some interesting properties of TE–TM mode-matching techniques are then demonstrated.**

I. Introduction

In the analysis of scattering from waveguide discontinuities, the mode-matching technique, whenever it can be applied, is by far the most popular way to solve the problem. It has been used to solve scattering problems in several different kinds of waveguides, such as the rectangular waveguide [1]–[5], the microstrip line [9], the finline [13], and the circular waveguide (or coaxial cable, see e.g. [18]). Scattering caused by the transition between different kinds of waveguides has been dealt with too (see e.g. [17]). Although several formulations can be used to represent the fields at the discontinuity interface, the TE–TM field expansion is the most general one when homogeneous waveguides with perfectly conducting walls are considered. Such field expansion derives the tangential components of the electric and magnetic fields from the longitudinal ones: the tangential components of the fields are then matched at the discontinuity interface, yielding a infinite system of linear equations. An approximate solution of the system of equations is then found by truncating the infinite series. Some properties of such approximate solution of the system of equations have been discussed in [14] (e.g. the relative convergence problem).

This paper focuses on some general properties of TE–TM field expansions in perfectly conducting waveguides when they are matched at some arbitrarily shaped waveguide discontinuity. It can be shown that the surface integrals resulting from matching the tangential field components can almost always be expressed as line integrals along the boundary of the region over which surface integration is carried out. Such reduction in the dimensionality of the integration is convenient from a computational point of view when the modes in the waveguides must be computed numerically. To be more specific, the line integral formulation is best suited when the modes in the waveguides are computed by techniques based on some integral equation expressed on the boundary, since in that case the modal eigenfunctions are computed only on the boundary of the waveguide cross section. The application of the line integral formulation of mode matching is then directly applicable, without the need for a time-consuming computation of the eigenfunctions at internal points in the waveguide cross section. On the other hand, the reduction to line integrals has pointed out an interesting property of mode-matching techniques: some coefficients representing coupling between TE and TM modes are always null; i.e., such modes are uncoupled. Such phenomenon have been observed in [2] and [17] for two particular cases (rectangular-to-rectangular waveguide junction and rectangular-to-circular waveguide junction). It will be shown here that it is quite general and independent of the shape of the waveguide cross section.

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II. Formulation

Consider the scattering problem represented by the transition between two arbitrarily shaped perfectly conducting walls waveguides (Fig. 1). Let S_1 be the cross section of guide 1, σ_1 its boundary, S_2 the cross section of guide 2, and σ_2 its boundary. Let then $\Omega = S_1 \cap S_2$ and C be its boundary.

Let $\boldsymbol{E}_{\mathrm{I}n}$ and $\boldsymbol{H}_{\mathrm{I}n}$ be the transverse electric and magnetic fields of the generic mode n in region I (with S_I the cross section of the related waveguide). Such tangential fields can be expressed as the sum of two contributions: a TE field and a TM field. They can be derived from the two longitudinal fields $E_{z\mathrm{I}n}$ and $H_{z\mathrm{I}n}$ (z being the coordinate relative to the direction of propagation, so that the z dependence of the fields is of the type $\exp(\mp j\beta z)$). By expanding the fields in the two waveguides as sums of the modal fields multiplied by unknown coefficients, one gets

$$\boldsymbol{E}_{\mathrm{I(TE)}} \cong \sum_{n}^{N_E} \left(e^{+}_{\mathrm{I(TE)}n} + e^{-}_{\mathrm{I(TE)}n}\right)\nabla\varphi_{\mathrm{I}n} \times \hat{z} \tag{1}$$

$$\boldsymbol{H}_{\mathrm{I(TE)}} \cong \sum_{n}^{N_E} \left(e^{+}_{\mathrm{I(TE)}n} - e^{-}_{\mathrm{I(TE)}n}\right)Y_{\mathrm{I(TE)}n}\nabla\varphi_{\mathrm{I}n} \tag{2}$$

$$\boldsymbol{E}_{\mathrm{I(TM)}} \cong \sum_{n}^{N_M} \left(e^{+}_{\mathrm{I(TM)}n} + e^{-}_{\mathrm{I(TM)}n}\right)\nabla\psi_{\mathrm{I}n} \tag{3}$$

$$\boldsymbol{H}_{\mathrm{I(TM)}} \cong \sum_{n}^{N_M} \left(e^{+}_{\mathrm{I(TM)}n} - e^{-}_{\mathrm{I(TM)}n}\right)Y_{\mathrm{I(TM)}n}\nabla\psi_{\mathrm{I}n} \times \hat{\boldsymbol{z}}. \tag{4}$$

Here

$$Y_{\mathrm{I(TE)}n} = \beta_{\mathrm{I(TE)}n} / \omega\mu$$

and

$$Y_{\mathrm{I(TM)}n} = \omega\epsilon / \beta_{\mathrm{I(TM)}n},$$

$\mathrm{Re}[\beta_{\mathrm{I(TE)}n}](\mathrm{Re}[\beta_{\mathrm{I(TM)}n}])$ being the propagation constant of the nth TE(TM) mode. Also, $\hat{\boldsymbol{z}}$ is the unit vector of the z axis, ∇ is the transverse gradient, ω is the angular frequency, μ is the magnetic permeability of the medium (throughout this work $\mu = \mu_0$), and ϵ is the dielectric constant. The modal series have been truncated by retaining only a limited number of modes. The unknown coefficients with suffixes "+" or "−" account for a wave traveling toward $+z$ (+) and a wave traveling toward $-z$ (−). The scalar functions $\varphi_{\mathrm{I}n}$ and $\psi_{\mathrm{I}n}$ are then the solu-